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16. Abstract

Analytical study of the problem of linear flutter of plane thin panels constructed symmetrically from an odd number of anisotropic layers. The supersonic gas flow in which the structure is placed is assumed to be coplanar and of arbitrary direction. After deriving the flutter equations, a criterion is presented for obtaining the critical flutter characteristics, and conclusions are drawn concerning the effect of various parameters taken into consideration.

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SUPERSONIC FLUTTER OF PLANE, RECTANGULAR, ANISOTROPIC HETEROGENEOUS STRUCTURES+

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1. Introduction /235*

In a number of recent studies of the aeroelastic stability of plane heterogeneous panels constructed symmetrically from orthotropic layers, it was assumed that:

a) the principal axes of orthotropy of the material of each layer coincides at every point with the panel's geometrical axes;

b) the panel is placed in a supersonic gas flow, and the velocity vector of the unperturbed flow is parallel to the direction $O\xi_1$.

In what follows, we shall analyze the problem of the flutter of plane heterogeneous structures constructed symmetrically from orthotropic layers, taking into consideration the effect of arbitrary orientation of the orthotropicity, as well as the effect of arbitrary orientation of the gas flow (which is assumed to be coplanar) on the panel's flutter characteristics.

2. Geometrical and Isotropic Considerations, Basic Equations

Let there be a plane rectangular plate (a x b), whose outer surface is exposed to a supersonic, coplanar, gas flow of arbitrary direction (Fig. 1 b).

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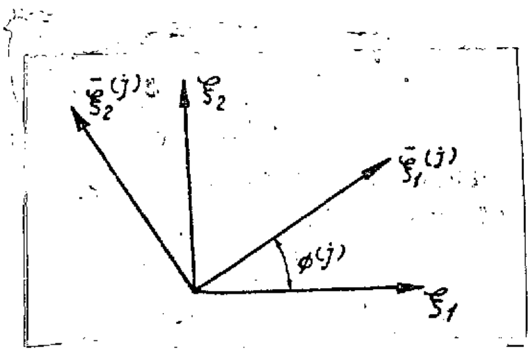

Fig. 1 a.

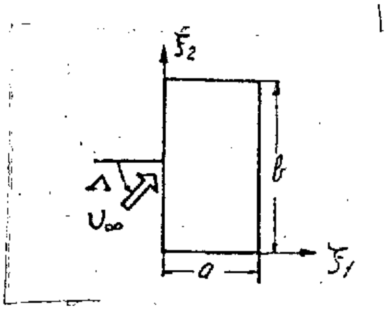

Fig. 1 b.

The panel is assumed to be constructed symmetrically from $2l + 1$ ($l = 1,2,\ldots$) elastic layers whose material is assumed to be homogeneous and orthotropic, and the principal axes $(\xi_1^{(j)}, \xi_2^{(j)})$ of the material of each layer are assumed to be rotated with respect to the geometrical axes (ξ_1, ξ_2) under the angle $\Phi^{(j)}$ (see Fig. 1 a)[1]. /236

But, as is well known (in this connection see, for example, [1]), in the case where the components of the tensor of the moduli of elasticity are referred to the panel's geometrical axes, which are assumed not to coincide with the principal axes of orthotropy, the material of each layer is characterized by six elastic constants, which corresponds to anisotropy of the elastic symmetry type with respect to the surface ξ_3 = const.

The method of Galerkin ceases to be an efficient instrument for tackling the various problems of the elastostatics and dynamics of plane anisotropic panels (homogeneous or heterogeneous; in this connection see Bert and Mayberry [2], Ashton [3] and Waddoups [4]), as well as the problem of the supersonic flutter of plane anisotropic panels (see Calligeros and Dugundji [5] and Ketter [6]).

On the other hand, the method of Rayleigh-Ritz, used in connection with the principle of the minimum of potential energy,

[1] The structure's geometrical and physical mechanical symmetry with respect to the panel's median plane also includes the values of the orientation angles of the orthotropicity of the material of the symmetric layers.

turns out to be a suitable and efficient instrument for tackling the above-mentioned problems. This is the method that we too shall use in analyzing the stated problem.

In this manner, adopting the hypothesis of Love-Kirchoff for the structure in the aggregate, the energy functional (see Ambartsumian [7]) is given by

$$Э = U - W, \tag{1}$$

where

$$\begin{aligned} U = \frac{1}{2}\int_0^a\int_0^b [&D_{11}(w_{,11})^2 + 2D_{12} w_{,11} w_{,22} + D_{22}(w_{,22})^2 + \\ &+ 4D_{66}(w_{,12})^2 + 4(D_{16} w_{,11} + D_{26} w_{,22})\, w_{,12} - \\ &- T_{11}^0 (w_{,1})^2 - T_{22}^0 (w_{,2})^2]\, d\xi_1\, d\xi_2 \end{aligned} \tag{2}$$

represents the total potential energy due to transversal bending /237
and the loads T_{11}^0 and T_{22}^0 in the plane of the plate (these are assumed to be positive in compression), and

$$W = \int_0^a\int_0^b pw\, d\xi_1\, d\xi_2 \tag{2'}$$

is the potential energy due to the transversal loads;

$$D_{ik} = \frac{2}{3}\sum_{j=1}^{l+1} B_{ik}^{(j)}(\xi_{(j)}^3 - \xi_{(j-1)}^3) \qquad (i, k = 1, 2, 6) \tag{3}$$

represents the bending strength, which can be expressed in terms of the constant $\overline{B}_{ik}$ (i, k = 1,2) referred to the principal axes $(\xi_1^{(j)}, \xi_2^{(j)})$ and the angles $\Phi^{(j)}$ with the aid of equations (5) from [1], adapted for the case of the theory of symmetrically constructed structures.

In the case of a supersonic, coplanar, gas flow of arbitrary direction Λ, the aerodynamic pressure will be given by

$$\Delta p = -\frac{\varkappa p_\infty}{c_\infty}\left[\frac{\partial w}{\partial t} + U_\infty\left(\frac{\partial w}{\partial \xi_1}\cos\Lambda + \frac{\partial w}{\partial \xi_2}\sin\Lambda\right)\right]. \tag{4}$$

Then the total transversal load $p(\xi_1,\xi_2,t)$ that is operative in the energy functional is expressed by

$$p(\xi_1, \xi_2, t) = -\frac{\varkappa p_\infty}{c_\infty}\left[\frac{\partial w}{\partial t} + U_\infty\left(\frac{\partial w}{\partial \xi_1}\cos\Lambda + \frac{\partial w}{\partial \xi_2}\sin\Lambda\right)\right] - m_0\varepsilon\frac{\partial w}{\partial t} - m_0\frac{\partial^2 w}{\partial t^2}, \tag{5}$$

In order to use the Rayleigh-Ritz method, we shall express the transversal shift $w(\xi_1,\xi_2,t)$ in the form

$$w(\xi_1, \xi_2, t) = \sum_{m,n} C_{mn} f_{mn}(\xi_1, \xi_2)\, e^{\omega t}, \tag{6}$$

where the modal functions $f_{mn}(\xi_1,\xi_2)$ must satisfy all the conditions at the kinematic limit.

In the case of a simple panel resting on its edge (the case to which we shall confine ourselves in the analysis that follows), the modal functions

$$f_{mn}(\xi_1, \xi_2) = \sin\frac{m\pi\xi_1}{a}\sin\frac{n\pi\xi_2}{b}, \tag{7}$$

permit the conditions at the kinematic limit to be adequately satisfied.

Taking into account equations (2)-(7) from [1], integrating and, in conformity with the principle of the minimum of /238 potential energy, imposing the condition

$$\frac{\partial Э}{\partial C_{mn}} = 0, \tag{8}$$

we obtain the following system of equations for the coefficients C_{mn}:

$$\begin{aligned}
&\left(\frac{D_{11}}{\overline{D}_{11}} m^4 + 2\varphi^2 \frac{D_{12}+2D_{66}}{\overline{D}_{11}} m^2 n^2 + \frac{D_{22}}{\overline{D}_{11}} n^4 \varphi^4 - R_{11} m^2 - R_{22} n^2 \varphi^2 - Z\right) C_{mn} - \\
&- \lambda \left[\sum_{p=1}^{\infty} \frac{4m(2p+1-m)}{(2p+1)(2p+1-2m)} C_{2p+1-m;\, n} \cos\Lambda + \right. \\
&\left. + \varphi \sum_{t=1}^{\infty} \frac{4n(2t+1-n)}{(2t+1)(2t+1-2n)} C_{m;\, 2t+1-n} \sin\Lambda \right] - \\
&- \frac{32}{\pi^2} \sum_{p=1}^{\infty} \sum_{t=1}^{\infty} \frac{mn(2p+1-m)(2t+1-n)}{(2p+1)(2t+1)(2p+1-2m)(2t+1-2n)} \times \\
&\times \left[(m^2 + (2p+1-m)^2) \frac{D_{16}}{\overline{D}_{11}} \varphi + (n^2 + (2t+1-n)^2) \frac{D_{26}}{\overline{D}_{11}} \varphi^3 \right] C_{2p+1-m;\, 2t+1-n} = 0,
\end{aligned} \tag{9}$$

where

$$R_{11} = \frac{T_{11}^0 a^2}{\pi^2 \overline{D}_{11}}, \qquad R_{22} = \frac{T_{22}^0 a^2}{\pi^2 \overline{D}_{11}}, \tag{10}$$

and

$-Z = \left(\frac{\omega}{\Omega_0}\right)^2 + \varepsilon_T \frac{\omega}{\Omega_0}$, is the parameter connected with the eigenvalues, (11)

$\lambda = \varkappa p_\infty M a^3/(\overline{D}_{11}\pi^4)$ is the velocity parameter,

$\Omega_0^2 = \pi^4 \overline{D}_{11}/(m_0 a^4)$ is the reference frequency

$\varepsilon_T = \varepsilon/\Omega_0 + \varkappa p_\infty/(m_0 c_\infty \Omega_0)$ is the total damping parameter.

With reference to these equations we can note the following:

a) in the case where $\Phi^{(j)} = 0$ and $\Lambda = 0$, the coupling of the modes in the direction $O\xi_1$ is purely aerodynamic;

b) in the case where $\Phi^{(j)} \neq 0$ and $\Lambda = 0$, the coupling of the modes can be of an aerodynamic and elastic nature (the elastic coupling taking place in the modes from the direction $O\xi_1$ as well as in those from the direction $O\xi_2$).

In the event that $\Lambda \neq 0$, present in addition is the coupling /239 of modes from the direction $O\xi_2$ that are operative in purely aerodynamic terms.

Also worthy of mention is the fact that the system of equations (9), particularized for different special cases, coincides with:

A) that obtained by Bohon [8] and Kordes and Noll [9] for the case of anhomogeneous and orthotropic panel (it being assumed that the elastic axes of orthotropy coincide with the geometrical axes) and the case of a homogeneous and isotropic panel, it being assumed in both cases that the panel is placed in a coplanar gas flow of arbitrary direction;

B) that obtained by Calligeros and Dugundji [5] for the case of ahomogeneous panel and a gas flow oriented parallel to the direction $O\xi_1$ ($\Lambda = 0$);

C) it agrees with the system of equations obtained by Ketter [6].

Returning to the system of flutter equations, the condition of nontriviality of the solution requires the determinant of the coefficients of C_{mn} to be zero.

Restricting our analysis to the case of the first two modes in the direction $O\xi_1$ as well as the direction $O\xi_2$ ($m = 1, 2$; $n = 1, 2$), the characteristic determinant becomes

$$\begin{vmatrix} \omega_{11}^2 - Z & -\frac{8\lambda}{3}\cos\Lambda & -\frac{8\lambda\varphi}{3}\sin\Lambda & \left[-\frac{640}{9\pi^2}\left(\frac{D_{16}}{\overline{D}_{11}}\varphi + \frac{D_{26}}{\overline{D}_{11}}\varphi^3\right)\right] \\ \frac{8\lambda}{3}\cos\Lambda & \omega_{21}^2 - Z & \left[\frac{640}{9\pi^2}\left(\frac{D_{16}}{\overline{D}_{11}}\varphi + \frac{D_{26}}{\overline{D}_{11}}\varphi^3\right)\right] & -\frac{8\lambda\varphi}{3}\sin\Lambda \\ \frac{8\lambda\varphi}{3}\sin\Lambda & \left[\frac{640}{9\pi^2}\left(\frac{D_{16}}{\overline{D}_{11}}\varphi + \frac{D_{26}}{\overline{D}_{11}}\varphi^3\right)\right] & \omega_{12}^2 - Z & -\frac{8\lambda}{3}\cos\Lambda \\ \left[-\frac{640}{9\pi^2}\left(\frac{D_{16}}{\overline{D}_{11}}\varphi + \frac{D_{26}}{\overline{D}_{11}}\varphi^3\right)\right] & \frac{8\lambda\varphi}{3}\sin\Lambda & \frac{8\lambda}{3}\cos\Lambda & \omega_{22}^2 - Z \end{vmatrix} = 0, \tag{12}$$

where

$$\omega_{mn}^2 = \frac{D_{11}}{\overline{D}_{11}} m^4 + 2\varphi^2 \frac{D_{12} + 2D_{66}}{\overline{D}_{11}} m^2 n^2 + \frac{D_{22}}{\overline{D}_{11}} n^4 \varphi^4 - R_{11} m^2 - R_{22} n^2 \varphi^2 \tag{13}$$

represents the natural frequencies obtained for damping and elastic coupling in the absence of a gas flow. /240

Expansion of the determinant leads to the following equation in the velocity parameter λ:

$$\begin{aligned}&\left(\frac{8\lambda}{3}\right)^4(\cos^2\Lambda-\varphi^2\sin^2\Lambda)^2+\left(\frac{8\lambda}{3}\right)^2\{\cos^2\Lambda[(\omega_{11}^2-Z)(\omega_{21}^2-Z)+\\&+(\omega_{12}^2-Z)(\omega_{22}^2-Z)]+\varphi^2\sin^2\Lambda[(\omega_{11}^2-Z)(\omega_{12}^2-Z)+\\&+(\omega_{22}^2-Z)(\omega_{21}^2-Z)]\}+(\omega_{11}^2-Z)(\omega_{22}^2-Z)(\omega_{12}^2-Z)(\omega_{21}^2-Z)+\\&+\frac{32^4\times20^4}{9^4\pi^8}\left(\frac{D_{16}}{\overline{D}_{11}}\varphi+\frac{D_{26}}{\overline{D}_{11}}\varphi^3\right)^4-\frac{32^2\times20^2}{9^2\pi^4}\left(\frac{D_{16}}{\overline{D}_{11}}\varphi+\frac{D_{26}}{\overline{D}_{11}}\varphi^3\right)^2\times\\&\times((\omega_{11}^2-Z)(\omega_{22}^2-Z)+(\omega_{12}^2-Z)(\omega_{21}^2-Z))+\\&+\left(\frac{8\lambda}{3}\right)^2\frac{2\times32^2\times20^2}{9^2\pi^4}\left(\frac{D_{16}}{\overline{D}_{11}}\varphi+\frac{D_{26}}{\overline{D}_{11}}\varphi^3\right)^2(\cos^2\Lambda+\varphi^2\sin^2\Lambda)-\\&-\left(\frac{8\lambda}{3}\right)^2\frac{64\times20}{9\pi^2}\varphi\left(\frac{D_{16}}{\overline{D}_{11}}\varphi+\frac{D_{26}}{\overline{D}_{11}}\varphi^3\right)\cos\Lambda\sin\Lambda\,((\omega_{11}^2-Z)+\\&+(\omega_{22}^2-Z)+(\omega_{12}^2-Z)+(\omega_{21}^2-Z))=0.\end{aligned}\qquad(14)$$

3. Formulating the Problem of Aeroelastic Stability

As is known, studying the aeroelastic stability of panels in a linear formulation permits determination of the value of the gas velocity called the flutter velocity, which is defined as follows: for the interval $0 \leq \lambda \leq \lambda_0$, the frequencies of ω satisfy the inequality $\mathrm{Re}_\omega \leq 0$ for which the solution of equation (6) is stable or neutrally stable while to the right of λ_0, there exist values of λ for which at least one solution of equation (6) possesses an ω that satisfies the inequality $\mathrm{Re}_\omega > 0$. In the latter case, the solution of equation (6) is unstable inasmuch as the panel undergoes oscillations of the flutter type.

It can be shown (in this connection see Movcian [10, 11] and Krumhaar [12]) that the determination of the conditions leading to definition of the critical flutter characteristics can be reduced to analysis of the problem's eigenvalues $Z_n \equiv Z_n(\lambda)$ at the limit as a function of the velocity parameter λ, the other parameters remaining constant.

For an eigenvalue $Z_n(\lambda)$[2] of the problem at the limit, we obtain from equation (11) two frequencies $\omega_{n1}(\lambda)$ and $\omega_{n2}(\lambda)$ given by

$$\omega_{n_{1,2}}(\lambda) = -\frac{1}{2}\varepsilon_T\Omega_0 \pm \sqrt{\left(\frac{1}{2}\varepsilon_T\Omega_0\right)^2 - \Omega_0^2 Z_n(\lambda)} \tag{15}$$

One of these two roots, namely, ω_{n1}, satisfies the inequality /241
$\mathrm{Re}(\omega_{n1}) < 0$ for an arbitrary λ; this inequality ensues from the fact that $\omega_{n1} + \omega_{n2} = -\varepsilon_p\Omega_0$. The second frequency, ω_{n2}, satisfies the conditions

$$\mathrm{Re}(\omega_{n2}) < 0 \quad \text{or} \quad \mathrm{Re}(\omega_{n2}) = 0 \quad \text{or} \quad \mathrm{Re}(\omega_{n2}) > 0 \tag{16}$$

if and only if $Z_n(\lambda)$ is located inside, at the boundary of or outside the stability parabola.

If the conditions under consideration are those that correspond to appearance of the threshold of instability ($\mathrm{Re}(\omega_{n2}) = 0$) and if we take into account that

$$Z = \mathrm{Re}\,Z + i\,\mathrm{Im}\,Z, \tag{17}$$

we get from equation (15)

$$\mathrm{Re}\,Z = \Omega_0^{-2}(\mathrm{Im}\,(\omega_{n_2}))^2, \qquad \mathrm{Im}\,Z = -\Omega_0^{-1}\varepsilon_T\,\mathrm{Im}(\omega_{n_2}). \tag{17'}$$

Equations (17') in the complex plane Z delimits the points of the parabola defined by

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[2] It should be stressed that for a value of $\lambda \geq 0$, the sequence of eigenvalues $Z_1(\lambda)$, $Z_2(\lambda)$, ... is denumerable.

$$Z_I^2 = \varepsilon_T^2 Z_R \qquad (Z_I \equiv \operatorname{Im} Z;\ Z_R \equiv \operatorname{Re} Z), \tag{18}$$

called the stability parabola[3].

The inner domain of the stability parabola corresponds to the eigenvalues for which the solutions of both ω_{n_1} and ω_{n_2} have real negative parts, and the outer domain corresponds to the eigenvalues for which $\operatorname{Re}(\omega_{n_2}) > 0$. Thus, the problem of determining the critical flutter velocity corresponding to the class of solutions of equation (6) reduces to analysis of the manner in which the problem's eigenvalues Z are ordered at the limit with respect to the stability parabola of equation (18).

The above considerations permit the proposed flutter problem to be studied. Thus, starting from the characteristic equation obtained, namely, equation (14), taking equation (17) into account in this equation and separating the real part from the imaginary part, we get two equations expressed through Z_R as well as through Z_P, equations that must be identically satisfied for the determinant of equation (11) to be zero.

On the other hand, as was mentioned earlier, the conditions for flutter to appear are obtained if and only if Z_R and Z_P are the coordinates of a point located on the stability parabola. Taking equation (18) into account in the above-mentioned equations, we get the system of equation expressed solely through Z_R, as /242
follows:

[3] The concept of stability parabola was introduced for the first time in the study of the aeroelastic stability of panels by Movcian [10], Krumhaar [12], and Stepanov [13] and used in the same field by Houbolt [14], Grigoliyuk and Mikhailov [15], Calligeros and Dugundji [15], Dowell [16], Ketter [6], and Vasiliyev [17]. In the wider context of the stability of non-conservative systems in general, the notion of stability parabola was introduced and generalized by Leipholz [18, 19].

$$\begin{aligned}
\mathrm{Re:}\ & \left(\frac{8\lambda}{3}\right)^4 (\cos^2\Lambda - \varphi^2 \sin^2\Lambda)^2 + \left(\frac{8\lambda}{3}\right)^2 \{\cos^2\Lambda[2(Z_R^2 - \varepsilon_T^2 Z_R) - \\
& - Z_R(\omega_{11}^2 + \omega_{12}^2 + \omega_{22}^2 + \omega_{21}^2) + \omega_{11}^2\omega_{21}^2 + \omega_{12}^2\omega_{22}^2] + \varphi^2 \sin^2\Lambda[2(Z_R^2 - \varepsilon_T^2 Z_R) - \\
& - Z_R(\omega_{11}^2 + \omega_{12}^2 + \omega_{22}^2 + \omega_{21}^2) + \omega_{11}^2\omega_{12}^2 + \omega_{22}^2\omega_{21}^2]\} + \\
& + \left(\frac{8\lambda}{3}\right)^2 \frac{2\times 32^2 \times 20^2}{9^2\pi^4} \left(\frac{D_{16}}{\overline{D}_{11}}\varphi + \frac{D_{26}}{\overline{D}_{11}}\varphi^3\right)^2 (\cos^2\Lambda + \varphi^2\sin^2\Lambda) - \\
& - \left(\frac{8\lambda}{3}\right)^2 \frac{64\times 20}{9\pi^2}\varphi\left(\frac{D_{16}}{\overline{D}_{11}}\varphi + \frac{D_{26}}{\overline{D}_{11}}\varphi^3\right)\sin\Lambda\cos\Lambda(\omega_{11}^2 + \omega_{22}^2 + \omega_{12}^2 + \\
& + \omega_{21}^2 - 4Z_R) + \frac{32^4\times 20^4}{9^4\pi^8}\left(\frac{D_{16}}{\overline{D}_{11}}\varphi + \frac{D_{26}}{\overline{D}_{11}}\varphi^3\right)^4 - \\
& - \frac{32^2\times 20^2}{9^4\pi^4}\left(\frac{D_{16}}{\overline{D}_{11}}\varphi + \frac{D_{26}}{\overline{D}_{11}}\varphi^3\right)^2 [2(Z_R^2 - \varepsilon_T^2 Z_R) - \\
& - Z_R(\omega_{11}^2 + \omega_{22}^2 + \omega_{12}^2 + \omega_{21}^2) + \omega_{11}^2\omega_{22}^2 + \omega_{12}^2\omega_{21}^2] + \\
& + Z_R^4 - 6\varepsilon_T^2 Z_R^3 + \varepsilon_T^4 Z_R^2 - (Z_R^3 - 3\varepsilon_T^2 Z_R^2)(\omega_{11}^2 + \omega_{22}^2 + \omega_{12}^2 + \omega_{21}^2) + \\
& + (Z_T^2 - \varepsilon_T^2 Z_R)[(\omega_{11}^2 + \omega_{21}^2)(\omega_{22}^2 + \omega_{12}^2) + \omega_{11}^2\omega_{21}^2 + \omega_{22}^2\omega_{12}^2] - \\
& - Z_R[\omega_{11}^2\omega_{21}^2(\omega_{22}^2 + \omega_{12}^2) + \omega_{12}^2\omega_{22}^2(\omega_{21}^2 + \omega_{11}^2)] + \omega_{11}^2\omega_{22}^2\omega_{12}^2\omega_{21}^2 = 0, \\
\mathrm{Im:}\ & \left(\frac{8\lambda}{3}\right)^2 (\cos^2\Lambda + \varphi^2\sin^2\Lambda)(4Z_R - \omega_{11}^2 - \omega_{22}^2 - \omega_{12}^2 - \omega_{21}^2) - \\
& - \frac{32^2\times 20^2}{9^2\pi^4}\left(\frac{D_{16}}{\overline{D}_{11}}\varphi + \frac{D_{26}}{\overline{D}_{11}}\varphi^3\right)(4Z_R - \omega_{11}^2 - \omega_{22}^2 - \omega_{12}^2 - \omega_{21}^2) + \\
& + \left(\frac{8\lambda}{3}\right)^2 \frac{64\times 80}{9\pi^2}\varphi\left(\frac{D_{16}}{\overline{D}_{11}}\varphi + \frac{D_{26}}{\overline{D}_{11}}\varphi^3\right)\cos\Lambda\sin\Lambda + \\
& + 4Z_R^3 - 4Z_R^2\varepsilon_T^2 + 2Z_R[(\omega_{11}^2 + \omega_{21}^2)(\omega_{22}^2 + \omega_{12}^2) + \omega_{11}^2\omega_{21}^2 + \\
& + \omega_{22}^2\omega_{12}^2] - (3Z_R^2 - Z_R\varepsilon_T^2)(\omega_{11}^2 + \omega_{12}^2 + \omega_{21}^2 + \omega_{22}^2) - \\
& - [\omega_{11}^2\omega_{21}^2(\omega_{12}^2 + \omega_{22}^2) + \omega_{12}^2\omega_{22}^2(\omega_{11}^2 + \omega_{21}^2)] = 0.
\end{aligned} \tag{19}$$

Any value of Z_R that satisfies both parts of equation (19) /243 corresponds to a flutter point. The criterion of dynamic instability therefore reduces to the condition that the system of equations (19) have at least one common root, a condition that is satisfied if and only if Sylvester's determinant, consisting of the coefficients of the above equations, is zero. An approximate method of obtaining the common roots of the two equations is the graphic method, which consists in representing the two polynomials in the plane of the parameters (Z_R, λ) and in choosing the point of intersection corresponding to the most critical flutter case.

In the case of damping $\varepsilon_T \approx 0$, the parameter Z that plays the role of the eigenvalues is expressed through

$$Z = -\left(\frac{\omega}{\Omega_0}\right)^2 \tag{20}$$

and the conditions that correspond to incipient flutter reduce to $\mathrm{Im}Z \cong 0$

In this case, instead of the system of equations (19), we get

$$\begin{aligned}
\mathrm{Re}: &\left(\frac{8\lambda}{3}\right)^4 (\cos^2\Lambda - \varphi^2 \sin^2\Lambda)^2 + \left(\frac{8\lambda}{3}\right)^2 \{\cos^2\Lambda[2Z_R^2 - Z_R(\omega_{11}^2 + \omega_{12}^2 + \\
&+ \omega_{21}^2 + \omega_{21}^2) + \omega_{11}^2\omega_{21}^2 + \omega_{12}^2\omega_{22}^2] + \varphi^2 \sin^2\Lambda[2Z_R^2 - \\
&- Z_R(\omega_{11}^2 + \omega_{12}^2 + \omega_{21}^2 + \omega_{21}^2) + \omega_{11}^2\omega_{12}^2 + \omega_{22}^2\omega_{21}^2]\} + \\
&+ \left(\frac{8\lambda}{3}\right)^2 \frac{2\times 32^2 \times 20^2}{9^2\pi^4} \left(\frac{D_{16}}{\overline{D}_{11}}\varphi + \frac{D_{26}}{D_{11}}\varphi^3\right)^2 (\cos^2\Lambda + \varphi^2\sin^2\Lambda) - \\
&- \left(\frac{8\lambda}{3}\right)^2 \frac{64\times 20}{9\pi^2} \varphi \left(\frac{D_{16}}{\overline{D}_{11}}\varphi + \frac{D_{26}}{\overline{D}_{11}}\varphi^3\right) \sin\Lambda \cos\Lambda(\omega_{1}^2 + \omega_{22}^2 + \\
&+ \omega_{12}^2 + \omega_{21}^2 - 4Z_R) + \frac{32^4\times 20^4}{9^4\pi^8} \left(\frac{D_{16}}{\overline{D}_{11}}\varphi + \frac{D_{26}}{\overline{D}_{11}}\varphi^3\right)^4 - \\
&- \frac{32^2\times 20^2}{9^2\pi^4} \left(\frac{D_{16}}{\overline{D}_{11}}\varphi + \frac{D_{26}}{\overline{D}_{11}}\varphi^3\right)^2 [2Z_R^2 - \\
&- Z_R(\omega_{11}^2 + \omega_{22}^2 + \omega_{12}^2 + \omega_{21}^2) + \omega_{11}^2\omega_{22}^2 + \omega_{12}^2\omega_{21}^2] + \\
&+ Z_R^4 - Z_R^3(\omega_{11}^2 + \omega_{22}^2 + \omega_{12}^2 + \omega_{21}^2) + Z_R^2[(\omega_{11}^2 + \omega_{21}^2)(\omega_{22}^2 + \omega_{12}^2) + \\
&+ \omega_{11}^2\omega_{21}^2 + \omega_{22}^2\omega_{12}^2] - Z_R[\omega_{11}^2\omega_{21}^2(\omega_{22}^2 + \omega_{12}^2) + \\
&+ \omega_{12}^2\omega_{22}^2(\omega_{21}^2 + \omega_{11}^2)] + \omega_{11}^2\omega_{22}^2\omega_{12}^2\omega_{21}^2 = 0, \\
\mathrm{Im}: &\left(\frac{8\lambda}{3}\right)^2 (\cos^2\Lambda + \varphi^2\sin^2\Lambda)(4Z_R - \omega_{11}^2 - \omega_{22}^2 - \omega_{12}^2 - \omega_{21}^2) - \\
&- \frac{32^2\times 20^2}{9^2\pi^4} \left(\frac{D_{16}}{\overline{D}_{11}}\varphi + \frac{D_{26}}{\overline{D}_{11}}\varphi^3\right)^2 (4Z_R - \omega_{11}^2 - \omega_{22}^2 - \omega_{12}^2 - \omega_{21}^2) +
\end{aligned} \tag{21}$$

$$+\left(\frac{8\lambda}{3}\right)^2 \frac{64\times 80}{9\pi^2}\varphi\left(\frac{D_{16}}{\overline{D}_{11}}\varphi+\frac{D_{26}}{\overline{D}_{11}}\varphi^3\right)\sin\Lambda\cos\Lambda+$$
$$+4Z_R^3+2Z_R[(\omega_{11}^2+\omega_{21}^2)(\omega_{22}^2+\omega_{12}^2)+\omega_{11}^2\omega_{21}^2+\omega_{22}^2\omega_{12}^2)]-$$
$$-3Z_R^2(\omega_{11}^2+\omega_{12}^2+\omega_{21}^2+\omega_{22}^2)-[\omega_{11}^2\omega_{21}^2(\omega_{12}^2+\omega_{22}^2)+$$
$$+\omega_{12}^2\omega_{22}^2(\omega_{11}^2+\omega_{21}^2)]=0,$$

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a system that is characterized by the fact that equation $(21)_2$ can be obtained in an exact manner from equation $(21)_1$ by derivation with respect to Z_R. This brings to light the fact that, in the event that $\varepsilon_T \approx 0$, the critical flutter parameters corresponding to simultaneous solution of the two equations correspond to the peak of the curve representing the variation of λ as a function of Z_R or, what comes to the same thing, the point at which the two branches of the natural frequency spectrum fuse.

Numerical Application, Conclusions

The effect of the variation in the angle of orientation of the orthotropicity on the critical flutter values will be brought to light for the case of a plane panel constructed symmetrically from three orthotropic layers, it being assumed that the outer layers have the thickness $h_1 = h_3 = \delta$ and the middle layer has the thickness $h_2 = 4\delta$ (total thickness $h = 4\delta$) [sic]. The outer layers are considered to be constructed from a material with orthotropy of type 1 and the middle layer, from a material with orthotropy of type 2 (see Table 1).

TABLE 1

Type of orthotropy	$\overline{E}_1$	$\overline{E}_2$	$\overline{\mu}_1$	$\overline{\mu}_2$	$\overline{G}_{12}$
Type 1	E	$10E$	0,0349	0,349	0,5
Type 2	$10E$	E	0,349	0,0349	0,5

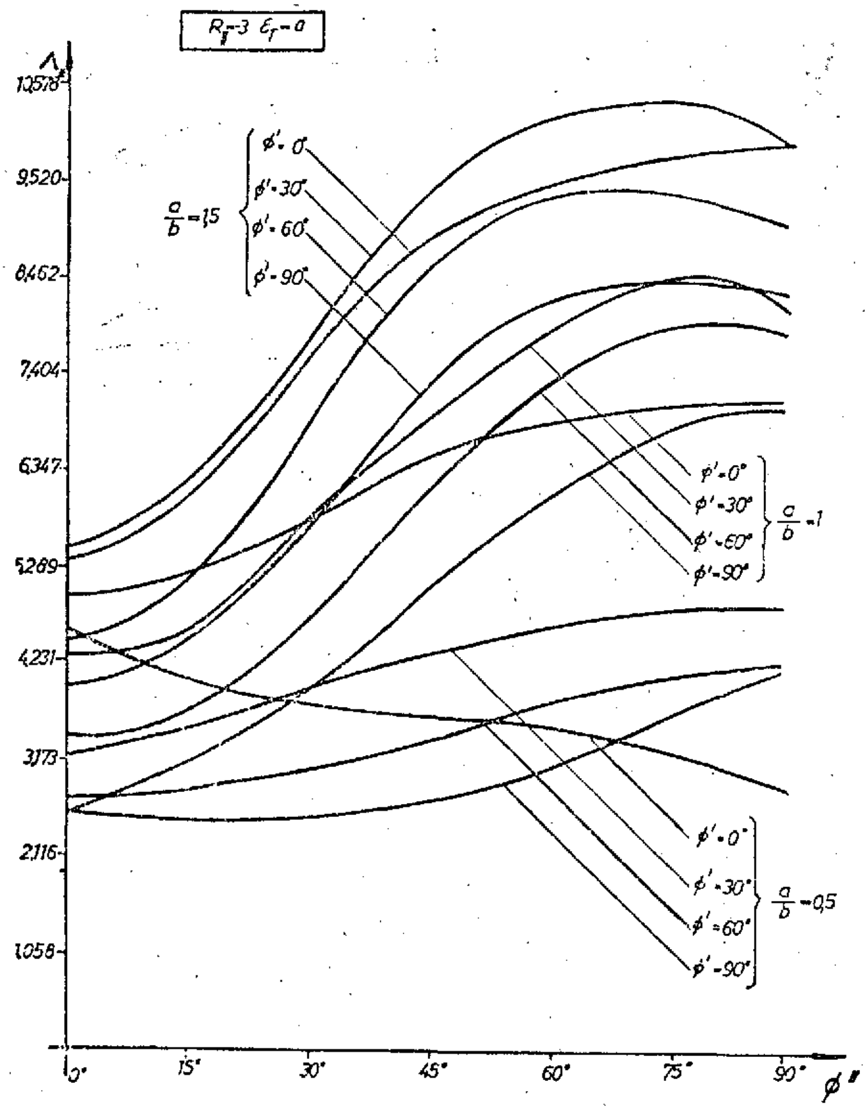
As regards the angles of orientation of the orthotropicity corresponding to the material of the three layers, it is considered to be $\Phi^{(2)} = \Phi^{(3)} \equiv \Phi''$; $\Phi^{(1)} \equiv \Phi'$.

The direction of flow of the gas is considered to be parallel with the axis $O\xi_1$.

In the case of the example under consideration, we get the variation of the flutter velocity λ as a function of the angle of orthotropicity Φ'' for different values of the angle Φ' and different values of the parameters ϕ, R_{11}, ε_T.

The main conclusion to be drawn from the curves obtained is that the maximum value of the critical flutter velocity does not arise in the general case when the principal axes of orthotropy coincide with the panel's geometrical axes (in this connection, see Figs. 2-6). This fact reflects the potential capability of these structures to bring about optimum conditions from the point of view of flutter requirements.

The same curves bring to light an increase in the critical flutter velocity that corresponds to the increase in the ratio $\phi = a/b$; the same thing happens in the case of an increase in the damping parameter ε_T. As regards the effect of R_{11} on the critical flutter velocity, Figs. 2 and 4 and, respectively, Figs. 2 and 3, show that while an increase in stretching loads ($R_{11} < 0$) is favorable, the effect of an increase in compression loads ($R_{11} > 0$) is unfavorable. Fig. 6 contains representations of the frequency curves that bring to light the points corresponding to the most critical flutter conditions, points obtained from the intersection of the curves resulting from representation of equations $(19)_1$ and $(19)_2$. /250



$R_{\mathrm{T}}=3$ $\varepsilon_T=0$
Λ
10,578
9,520
8,462
7,404
6,347
5,289
4,231
3,173
2,116
1,058
$\frac{a}{b}=1,5$
$\phi'=0°$
$\phi'=30°$
$\phi'=60°$
$\phi'=90°$
$\frac{a}{b}=1$
$\phi'=0°$
$\phi'=30°$
$\phi'=60°$
$\phi'=90°$
$\frac{a}{b}=0,5$
$\phi'=0°$
$\phi'=30°$
$\phi'=60°$
$\phi'=90°$
0°
15°
30°
45°
60°
75°
90°
ϕ''

Fig. 2.

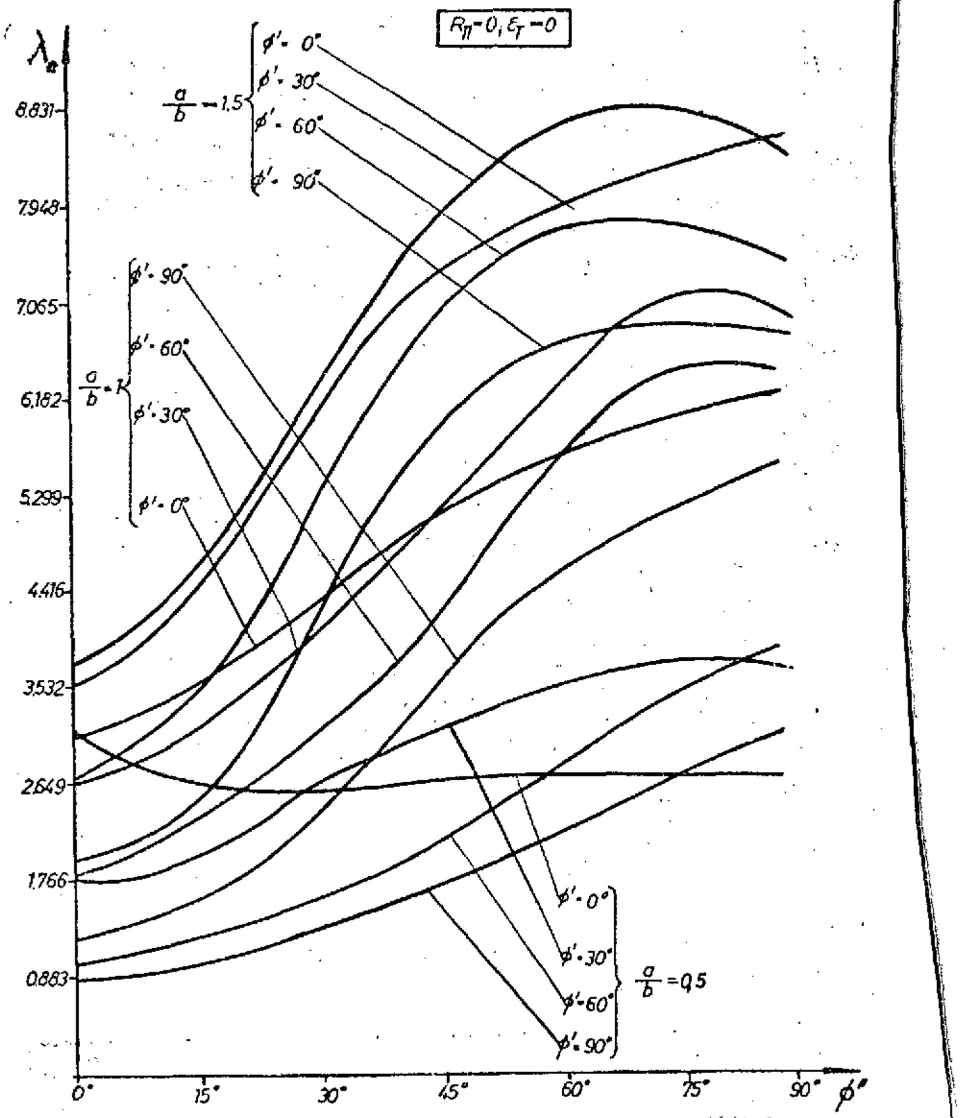

$R_{II}=0, \varepsilon_T=0$
λ_e
8.831
7.948
7.065
6.182
5.299
4.416
3.532
2.649
1.766
0.883
$\frac{a}{b}=1.5$
$\phi'=0°$
$\phi'=30°$
$\phi'=60°$
$\phi'=90°$
$\frac{a}{b}=1$
$\phi'=90°$
$\phi'=60°$
$\phi'=30°$
$\phi'=0°$
$\frac{a}{b}=0.5$
$\phi'=0°$
$\phi'=30°$
$\phi'=60°$
$\phi'=90°$
0°
15°
30°
45°
60°
75°
90°
$\phi°$

Fig. 3.

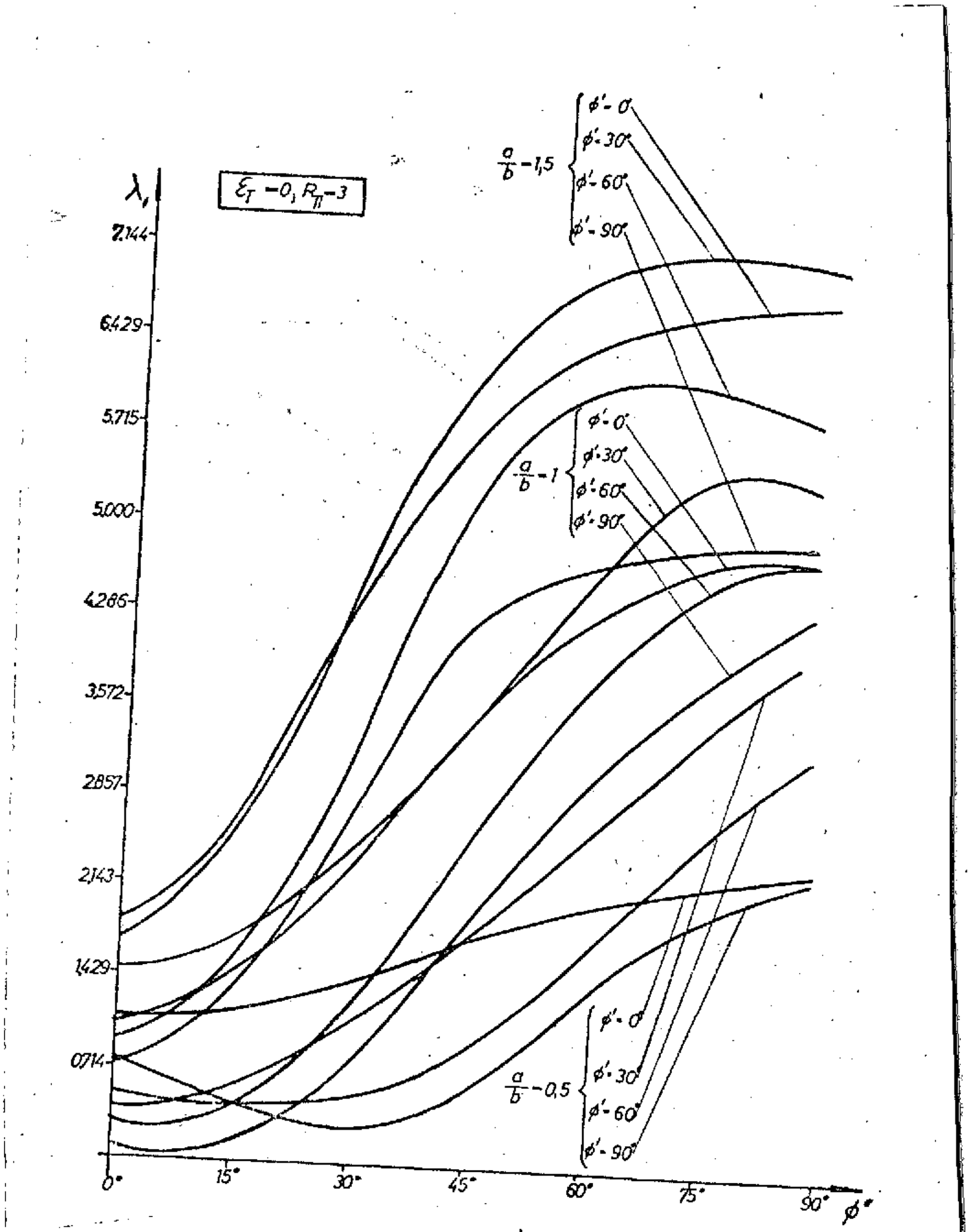
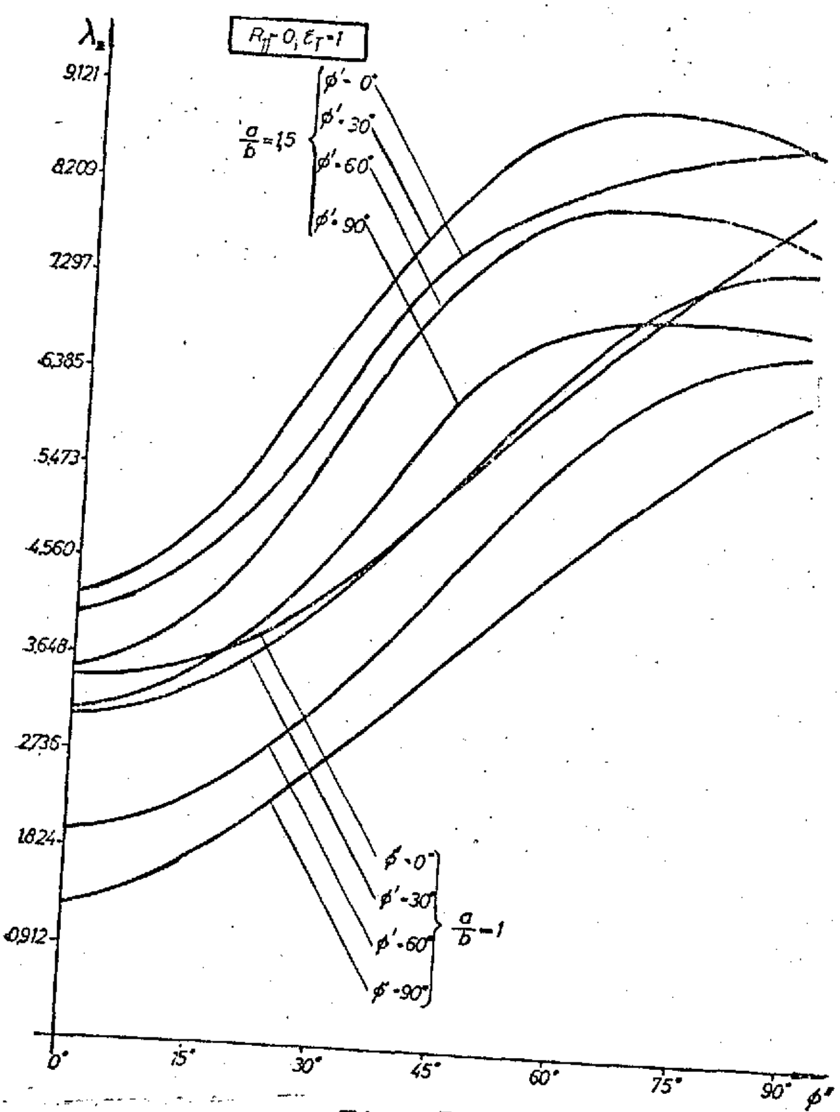

$\varepsilon_T = 0, R_{\Pi} = 3$
λ_1
7.144
6.429
5.715
5.000
4.286
3.572
2.857
2.143
1.429
0.714
$\frac{a}{b} = 1.5$
$\phi' = 0°$
$\phi' = 30°$
$\phi' = 60°$
$\phi' = 90°$
$\frac{a}{b} = 1$
$\phi' = 0°$
$\phi' = 30°$
$\phi' = 60°$
$\phi' = 90°$
$\frac{a}{b} = 0.5$
$\phi' = 0°$
$\phi' = 30°$
$\phi' = 60°$
$\phi' = 90°$
0°
15°
30°
45°
60°
75°
90°
$\phi°$

Fig. 4.



λ_z
$R_{II}=0, \varepsilon_T=1$
9,121
8,209
7,297
6,385
5,473
4,560
3,648
2,736
1,824
0,912
$\frac{a}{b}=1,5$
$\phi'=0°$
$\phi'=30°$
$\phi'=60°$
$\phi'=90°$
$\frac{a}{b}=1$
$\phi'=0°$
$\phi'=30°$
$\phi'=60°$
$\phi'=90°$
0°
15°
30°
45°
60°
75°
90°
ϕ''

Fig. 5.

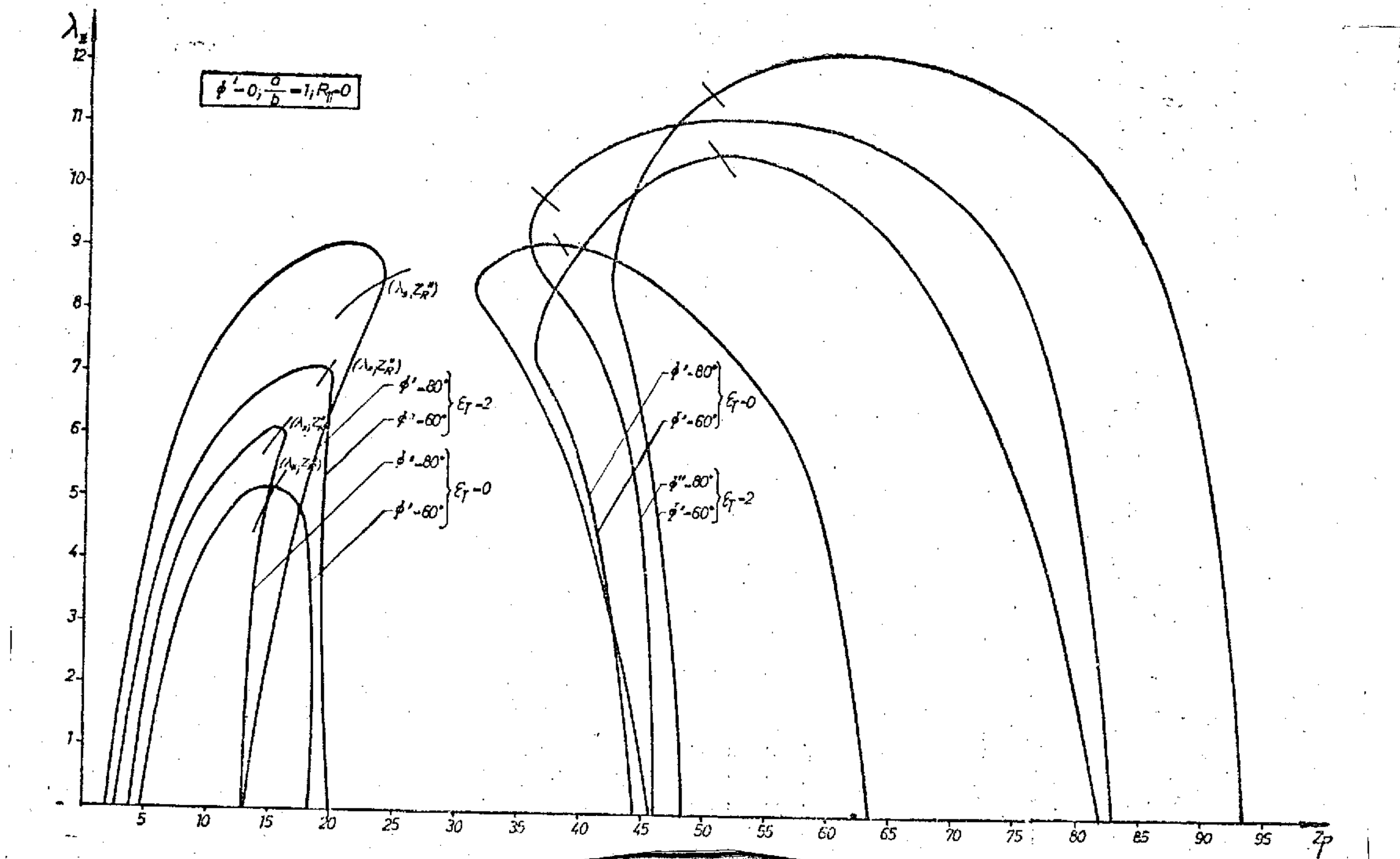

$\phi' = 0;\ \frac{a}{b} = 1;\ R_{II} = 0$
λ_{II}
(λ_s, Z_R^*)
$\phi' = 80°$
$\phi' = 60°$
$\varepsilon_T = 2$
$\varepsilon_T = 0$
Z_P
5
10
15
20
25
30
35
40
45
50
55
60
65
70
75
80
85
90
95
1
2
3
4
5
6
7
8
9
10
11
12

Fig. 6.

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REFERENCES

1. Librescu, L. and Badoiu, Tr., "The effect of the orientation of orthotropicity in the problem of the supersonic flutter of thin, heterogeneous, cylindrical, circular structures of infinite length," St. Cerc. Mec. Apl. 31, 5 (1971).

2. Bert, C.W., and Mayberry, B.L., "Free vibrations of unsymmetrically laminated anisotropic plates with clamped edges," J. Composite Materials 3, 282-293 (1969).

3. Ashton, J.E., "Analysis of anisotropic plates II," J. Composite Materials 3, 470-479 (1969).

4. Ashton, J.E. and Waddoups, M.E., "Analysis of anisotropic plates," J. Composite Materials 3, 148-165 (1969).

5. Calligeros, J.M and Dugundji, J., "Supersonic flutter of rectangular orthotropic panels with arbitrary orientation of orthotropicity," APOSR TR 5328, 1963.

6. Ketter, D.J., "Flutter of flat, rectangular, orthotropic panels," AIAA Journal 5(1), 116-124 (1967).

7. Ambartsumian, S.A., Teoriya anizotropnykh oboloschek [The Theory of Anisotropic Plates], "Fizmatgiz" Press, Moscow, 1961.

8. Bohon, H.L., "Flutter of flat rectangular orthotropic panels with biaxial loading and arbitrary flow directions," NASA TN D-1949, 1963.

9. Kordes, E.E. and Noll, R.B., "Theoretical flutter analysis of flat rectangular panels in uniform coplanar flow with arbitrary direction," NASA TN D-1156, 1962.

10. Movchan, A.A., "On the vibrations of plates moving in a gas," Priklad. Matem. i Mekhan. 20, 2 (1956).

11. Movchan, A.A., "On the stability of panels moving in a gas," Priklad. Matem. i Mekhan. 21, 2 (1957).

12. Krumhaar, H., "Supersonic flutter of circular cylindrical shell of finite length in an axisymmetrical mode," Int. J. Solids Structures 1, 23-57 (1965).

13. Stepanov, R.D., "On the flutter of cylindrical plates and panels moving in a gas flow," Priklad. Matem. i Mekhan. 21, 5 (1957).

14. Houbolt, J.C., "A study of several aerothermoelastic problems of aircraft structures in high-speed flight, 5," Mitt. Inst. Flugzeugstatik Leichthan, Leeman (Zurich), 1958.

15. Grigoliyuk, Ye.I., and Mikhailov, A.P., "Flutter of a three-layered, circular, conical plate," Dokl. AN SSSR 163(5), 1100-1103 (1965).

16. Dowell, E.H.,"Flutter of multibay panels at high supersonic speeds," AIAA Journal 2, 10 (1969).

17. Vasiliyev, Yu. V., "Supersonic flutter of cylindrical layers of plates," Rev. Roum. Sci. Techn. Mec. Appl. 15, 4 (1970).

18. Leipholz, H., "On the influence of damping in nonconservative problems of the stability of elastic bars," Ingenieur-Archiv 33(5), 308-321 (1969).

19. Leipholz, H., "Outline of a theory of stability for elastic systems under nonconservative loading," Ingenieur-Archiv 4(1), 56-68 (1965).